Int. J. Heat Mass Transfer. Vol. 11, pp. 1727–1730. Pergamon Press 1968. Printed in Great Britain

SHORTER COMMUNICATION

EXPLICIT SOLUTIONS OF THE NUSSELT RELATIONS FOR LAMINAR CONDENSATION

E. J. HOFFMAN

National Resources Research Institute, The University of Wyoming, Laramie, Wyoming 82070, U.S.A.

(*Received* 9 *January* 1968 *and in revised form* 22 *April* 1968)

NOMENCLATURE

a, b, constants for curve fit;
D, tube diameter;
g, acceleration of gravity;
h_c, condensing coefficient;
h_N, mean condensing coefficient for N tubes;
h_0, convection coefficient;
k, thermal conductivity of condensate;
L, tube length;
n_i, number of horizontal tubes in a vertical row "i";
n_e, effective number of vertical rows;
n_a, average number of tubes per vertical row;
N, total number of tubes;
R_{F_0}, fouling resistance;
R_{w_0}, wall resistance;
T_v, vapor temperature;
T', coolant temperature;
ΔT, temperature drop across condensate film;
U', heat-transfer coefficient between condenstate-tube wall interface and coolant;
Δx, distance between vertical centerlines for vertical rows;
Δy, distance between tube centers in a vertical row.

Greek symbols
Γ, linear condensate rate;
λ, latent heat;
μ, viscosity of condensate;
ρ, density of condensate;
ϕ, symbol for grouping containing overall temperature difference;
ψ, symbol for grouping containing condensate properties.

THE VISCOUS relation for laminar flow down surfaces, when used to calculate the film thickness and applied to an energy balance, leads to the classic Nusselt expressions for laminar condensation. These relationships are usually presented as the condensing heat-transfer coefficient, in either the third-root or fourth-root forms. The application to design ordinarily requires trial-and-error procedures, and it is the purpose here to develop shortcut procedures.

For vertical tubes or surfaces, solution for the mean heat-transfer coefficient leads to either the third- or fourth-root forms, which may be combined to eliminate the heat-transfer coefficient, giving:

$$\Gamma^{\frac{4}{3}} = \frac{\frac{4}{3}}{3^{\frac{1}{3}}}\left[\frac{k^3\rho^2 g}{\mu}\right]^{\frac{1}{3}}\frac{\Delta TL}{\lambda}. \tag{1}$$

A similar reduction for an array of horizontal tubes leads to

$$\Gamma^{\frac{4}{3}} = \frac{2^{\frac{1}{3}}(3{\cdot}428)}{3^{\frac{1}{3}}}\left[\frac{k^3\rho^2 g}{\mu}\right]^{\frac{1}{3}}\;\frac{\Delta TD}{\lambda}\;n_e^{\frac{1}{3}}N. \tag{2}$$

For an array of horizontal tubes, the effective number of vertical rows is defined by

$$n_e = \left[\frac{(\sum n_i^{\frac{3}{4}})^{\frac{4}{3}}}{N}\right]^3 \tag{3}$$

where n_i is the number of tubes in a vertical row i. A mean or average number of tubes per vertical row is defined by

$$n_a = \left[\frac{N}{\sum n_i^{\frac{3}{4}}}\right]^4. \tag{4}$$

Note that

$$n_e n_a = N = \sum n_i. \tag{5}$$

Use of the third-root forms for the condensing coefficient requires trial and error solution with Γ in the overall balance. Use of the fourth-root forms requires trial and error solution with ΔT. No economy of advantage is obtained over using the previously derived expressions for Γ—in fact an extra step is involved due to the introduction of the condensing coefficient as a variable.

It is possible to obtain a more explicit relation for Γ by eliminating ΔT as a variable. The energy rate balance to the coolant is, for vertical tubes

$$\lambda\Gamma = U'(T_w - T')L.$$

For horizontal tubes,

$$\lambda \Gamma = U'(T_w - T')N\Pi D$$

where

U' = transfer coefficient to the coolant;
T_w = tubewall temperature;
T' = coolant temperature.

Based on tube OD, for instance,

$$U' = \frac{1}{\frac{1}{h_0} + R_{w_0} + R_{F_0}}$$

where

h_0 = convection coefficient;
R_{w_0} = wall resistance;
R_{F_0} = fouling resistance.

If T_v is the vapor condensing temperature, then

$$\Delta T = T_v - T_w = (T_v - T') - (T_w - T')$$

and for vertical tubes or surfaces,

$$\Delta T = (T_v - T') - \frac{\lambda \Gamma}{U'L}$$

while for horizontal tubes,

$$\Delta T = (T_v - T') - \frac{\lambda \Gamma}{U'N\Pi D}.$$

Substituting for ΔT in the expressions for Γ gives the following abbreviated form upon rearranging:

$$\frac{\Gamma^{\frac{4}{3}}}{\phi - \Gamma} = \psi \qquad (6)$$

where

$$\phi = \frac{U'L}{\lambda}(T_v - T') \quad \text{for vertical tubes or surfaces}$$

$$= \frac{U'N\Pi D}{\lambda}(T_v - T') \quad \text{for horizontal tubes}$$

and

$$\psi = \frac{0{\cdot}924\left[\frac{k^3\rho^2 g}{\mu}\right]^{\frac{1}{3}}}{U'} \quad \text{for vertical tubes or surfaces}$$

$$= \frac{0{\cdot}9532\left[\frac{k^3\rho^2 g}{\mu}\right]^{\frac{1}{3}}_{n_e^{\frac{1}{3}}}}{U'} \quad \text{for horizontal tubes.}$$

Obviously, other arrangements could be formed. The equation can be reduced to two variables by the substitutions, where now

$$\Gamma' = \frac{\Gamma}{\psi^3}$$

and

$$\phi' = \frac{\phi}{\psi^3}$$

such that

$$\frac{(\Gamma')^{\frac{4}{3}}}{\phi' - \Gamma'} = 1. \qquad (7)$$

ALGEBRAIC SOLUTION TO QUARTIC

Equation (7) is the quartic

$$(\Gamma')^4 + (\Gamma')^3 + (-3\phi')(\Gamma')^2 + 3(\phi')^2\,\Gamma' + [-(\phi')^3] = 0. \qquad (7a)$$

Of the four roots, the proper root for the physical solution is [1]

$$\Gamma' = -a - (\sqrt{u}) + [\sqrt{(v + \sqrt{w})}] \qquad (8)$$

where

$$a = \tfrac{1}{4};$$

$$u = \tfrac{1}{16} + \frac{\phi'}{2} + f;$$

$$v = \tfrac{1}{8} + \phi' - f;$$

$$w = 4(\tfrac{1}{16} + \frac{\phi'}{2} + f)^2 + (\phi')^3 - 12\left[\tfrac{1}{16} + \frac{\phi'}{2}\right]f;$$

and where

$$f = \tfrac{1}{2}[\tfrac{1}{16}(\phi')^4 + \text{Rad}]^{\frac{1}{3}} + \tfrac{1}{2}[\tfrac{1}{16}(\phi')^4 - \text{Rad}]^{\frac{1}{3}}$$

if

$$\text{Rad} = (\phi')^4\left[1/256 + \frac{\phi'}{27}\right]^{\frac{1}{2}}.$$

The above constitutes an exact solution for Γ.

Knowing Γ, the line or dimension normal to the direction of condensate flow can be used to calculate the total condensate rate, W_L (or W_N). Thus, for vertical tubes,

$$W_L = \Gamma\,\pi N D.$$

For horizontal tubes,

$$W_L = \Gamma(L_2 - L_1)$$

where $(L_2 - L_1) \sim L$, the tube length.

Note that the condensing coefficient is not used. If of interest, it can be conveniently calculated from the third-root form, since Γ is known.

GRAPHICAL REPRESENTATION

The algebraic solution to the quartic is cumbersome except for computer applications. However, equations (6) or (7) can be conveniently plotted in advance for manual calculation purposes.

For instance, by arranging equation (6) as

$$\phi = \frac{\Gamma^{\frac{4}{3}} + \psi\Gamma}{\psi}$$

values of ϕ can be calculated from assumed values of Γ for parameters of ψ. This can be plotted as Γ vs. ϕ for the parameter ψ. Such a plot is shown in the accompanying figure for limited ranges of ϕ and Γ.

Equation (7) arranges to

$$\phi' = (\Gamma')^{\frac{4}{3}} + \Gamma'$$

from which a plot may also be conveniently made.

For a given value of ψ, the accompanying plot indicates that over an interval there is an approximate linear log–log relation between Γ and ϕ. This is also the situation for the behaviour of Γ' vs. ϕ'. Thus, over an interval,

$$\Gamma' \sim a(\phi')^b$$

$$\Gamma \sim a\psi^{3(1-b)}\phi^b.$$

The exponent b can be considered to vary as ϕ' approaches zero or becomes infinite. The intercept a will have a minimum of about 0·55 at $\phi' = 1$, and vary to values of unity as $\phi' \to 0$ or $\phi' \to \infty$. In fact, a and b can be fit to functions of ϕ' to obtain a curve fit for Γ' vs. ϕ', or Γ vs. ϕ.

Note that if $b = 1$,

$$\Gamma = a\phi$$

$$= a\frac{U'L}{\lambda}(T_v - T') \quad \text{for vertical tubes or surfaces}$$

$$= a\frac{U'N\pi D}{\lambda}(T_v - T') \quad \text{for horizontal tubes.}$$

Hence there exists some overal coefficient U between T_v and T' such that

$$U = aU'$$

or

$$\frac{1}{U} = \frac{1}{aU'} = \frac{1}{h_c} + \frac{1}{U'}$$

where h_c is the condensing coefficient. Solving for h_c,

$$h_c = \frac{1}{U'}\left(\frac{1}{a} - 1\right).$$

This result effectively points up that the behaviour of the condensing coefficient is affected by the other resistances or conductances involved. This relationship appears much more obscure in the rigorous form, but is there, nevertheless.

EXAMPLE

To demonstrate the direct solution for the condensate rate without employing the condensing coefficient concept, certain portions will be utilized from a problem in [2]

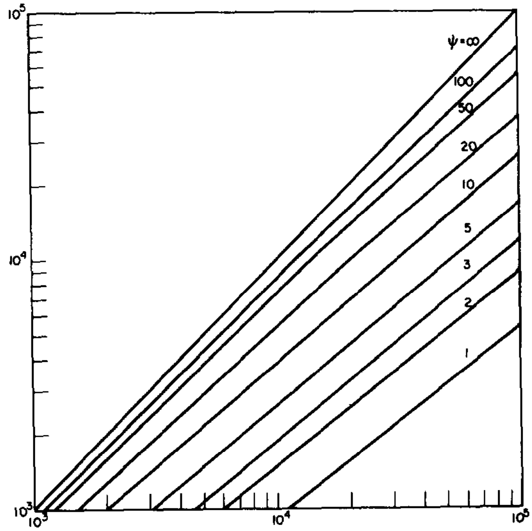

FIG. 1.

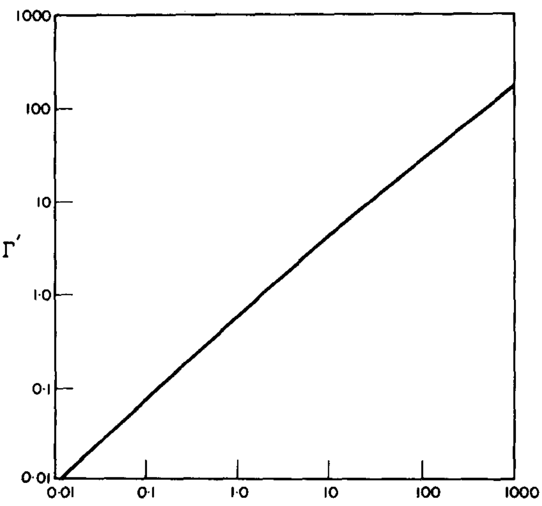

FIG. 2.

In the problem statement, n-propanol is to be condensed on the tube side of a horizontal condenser, at the rate of 60000 lb/h. It is agreed to use 766 tubes of $\frac{3}{4}$ in O.D. in $\frac{15}{16}$ in triangular pitch, with four tube passes. Appearing, or calculated from the data, are the following values:

Temperatures.

$$T_v = 244°\text{F}$$

$$T_v - T' = LMTD = 141°\text{F}.$$

Physical properties of the condensate.

$$\mu \sim 0{\cdot}62 \text{ c.p.}$$

$$k \sim 0{\cdot}094 \text{ Btu/h ft}^2 \text{ °F/ft}$$

$$\rho \sim 0{\cdot}80\,(62{\cdot}5) = 50 \text{ lb/ft}^3$$

$$\lambda \sim 285 \text{ Btu/lb}.$$

Properly, the condensate film properties should be evaluated at some average defined film temperature. However, a temperature based on the $LMTD$ could as well be used, since the calculation has a measure of approximation.

Coolant to wall resistances

$$h_0 = 1075 \text{ Btu/h ft}^2 \text{ °F};$$

R_w, assumed negligible;

R_f, assumed negligible.

Therefore, $U' \sim 1075$.

Bundle characteristics

$$N = 766;$$

$$D = 0{\cdot}0625 \text{ ft}.$$

A value for n_a may be found from [3]. For 766 tubes on a triangular pitch, $n_a \sim 13$.

A value for n_a can also be calculated from the limiting formula for circular tube fields:

$$n_a = N^{\frac{1}{2}} \frac{(\sqrt{\pi})}{2} \left(\frac{\Delta x}{\Delta y}\right)^{\frac{1}{2}}$$

or

$$\ln n_a = \tfrac{1}{2} \ln N + \ln \left[\frac{(\sqrt{\pi})}{2} \left(\frac{\Delta x}{\Delta y}\right)^{\frac{1}{2}}\right]$$

where Δx and Δy are the distances between tube centers in horizontal and vertical rows, respectively. For a triangular layout,

$$\frac{\Delta x}{\Delta y} = \frac{1}{4 \sin 60°}.$$

Thus

$$n_a = (766)^{\frac{1}{2}} \frac{(\sqrt{\pi})}{2} \left[\frac{1}{4\,(0{\cdot}87)}\right]^{\frac{1}{2}} = 13{\cdot}3.$$

Since $N = n_e n_w$

$$n_e = \frac{766}{13} \sim 60.$$

Calculation of ϕ, ψ, and Γ

$$\phi = \frac{U' N \pi D}{\lambda} (T_v - T')$$

$$= \frac{1075(766)\,\pi(0{\cdot}0625)}{285} (141)$$

$$= 80000$$

$$\psi = \frac{0{\cdot}9532 \left[\dfrac{k^3 \rho^2 g}{\mu}\right]^{\frac{1}{3}} n_e^{\frac{1}{3}}}{U'}$$

$$= \frac{0{\cdot}9532 \left[\dfrac{(0{\cdot}094)^3\,(50)^2\, 4{\cdot}18 \times 10^8}{0{\cdot}62\,(2{\cdot}42)}\right]^{\frac{1}{3}}}{1075} 60^{\frac{1}{3}}$$

$$= 2{\cdot}91.$$

Alternately

$$\phi' = \frac{\phi}{\psi^3} = \frac{80000}{(2{\cdot}91)^3} = 3250.$$

From a plot of Γ vs. ϕ and ψ.

$$\Gamma \sim 9400 \text{ lb/h}.$$

To condense 60000 lb/h of n-proponol, the condenser should be 60000/9400 $\sim$ 6 ft long. The total length assigned in Kern [2] was 8 ft.

Determination of the condensation coefficient. Though not necessary, the value of the condensing coefficient can now be calculated:

$$h_c \,(\text{or } h_N) = 0{\cdot}9532 \left[\frac{k^3 \rho^2 g}{\mu}\right]^{\frac{1}{3}} \left(\frac{1}{\Gamma}\right)^{\frac{1}{3}} n_e^{\frac{1}{3}}$$

$$= 0{\cdot}9532\,(840) \left(\frac{1}{9400}\right)^{\frac{1}{3}} 60^{\frac{1}{3}}$$

$$= 148 \text{ Btu/h ft}^2 \text{ °F}.$$

The main discrepancy with the Kern result ($h_c \sim 172$ Btu/h ft^2 °F) is in that a value for n_e was assigned equal to $N^{\frac{2}{3}} = 83$, rather than $n_e = 60$. This adjustment brings both results into line.

REFERENCES

1. M. Merriman, *Solution of Equations*, 4th edn. John Wiley, New York (1906).
2. D. Q. Kern, *Process Heat Transfer*, pp. 274–277. McGraw-Hill, New York (1950).
3. A. Devore, How to design multitube condensers, *Petrol. Refiner* **38**(6), 205–216 (1959).